Diagram design method to enhance mathematical learning

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Abstract

An exploration of current design practice found that educational publishers may not spend sufficient resources on the design and evaluation of mathematical diagrams. In appropriate and misleading diagrams may contribute to problems in the teaching and learning of mathematics. The design of such diagrams may be led by aesthetic as opposed to pedagogic considerations, against a need to produce material cheaply and quickly, without user consultation or evaluation. In response to these findings, a design method -Mathematical Diagram Design Method (MDDM) was developed to ensure the design of effective mathematical diagrams, augmented by cognitive and perceptual design guidelines for designers. MDDM aims to build up designer's understanding of the task, user, tool and environment (TUTE). The method is based on the generation of explicit requirements, elicited by the designer during focused discussions with the teacher (who may act as both domain expert and representative for the end users). The mathematical specifications lead to the co-creation of conceptual sketches by the designer and teacher, which ensure a transfer of knowledge and that the designer understands the mathematical process to be conveyed in the diagrams. Although the diagrams produced using this method did not show a significant quantitative improvement in students' learning, teachers believed that the diagrams would improve teaching and encourage students to reflect on the underlying concept – this was shown qualitatively. Educational diagram designers believed such a method would enhance current design practice.

Keywords: Diagram, Design method, Children, Information design, Mathematical teaching and learning.

1. Introduction

A worrying number of children and adolescents in the UK demonstrate difficulty in understanding mathematics in general and geometrical concepts [3, 10] in particular. Young students in the UK are failing to learn basic geometric concepts and geometric problem solving such. As a consequence, they are under-prepared for the study of more sophisticated concepts, especially compared to students from Japan and America [9].

Researchers [10, 17] have suggested that the failure to understand geometry (Shape and Space) in young children may be due to the perceived poor quality of the teaching and learning materials, including diagrams. Poorly designed diagrams have been shown to reduce the effective teaching and learning of mathematics [10, 16, 18]. Typical problems include data not being organized to represent fundamental ideas or steps so as to underline the mathematical concepts involved, and failure of diagrams to support spatial and visual reasoning [10, 14]. Also research

[13, 17] has shown that if inaccurate diagrams are used in classrooms these can confuse children who are struggling to learn new concepts. Clear, unambiguous, effective diagrams are needed to support the teaching of concepts in geometry. Therefore, a method is needed to enable diagrams to be created which facilitate visual reasoning and which associate mathematical language with Shape and Space.

It is known that diagram comprehension is part of a larger cognitive process involving perceptual pattern finding and cognitive symbolic operations [12, 14]. Thus a diagram's effectiveness, to some extent, depends on how well it is designed as an input to the perceptual system. It is important to apply instructional design and perceptual principles into the design guideline for designers [13, 14].

Professional designers and teachers interviewed indicated that typically educational material is not designed in a systematic manner, often without evaluation with teachers or students [4, 8]. To improve on this, research [11, 15, 20] suggests that the development should include participation with users. Researchers in educational software [7, 15] proposed that individual stakeholders involved in the project development should explicitly describe what they think is important and what they can contribute. Their proposition is helpful in highlighting the need for including key stakeholders but failed to consider the role of the designer in this process. In addition, information theorists suggest that designers must understand users and requirements in order to direct user's attention to key components essential and relevant to the task [6, 11]. However, often designers do not have access to users or a clear specification. Without these, designers have difficulty in forming design goals, solutions, and schematizations on the users' comprehension [20]. Hence, a systematic design method with user centred approach and designer participation during the product development is required to enhance diagram quality.

2. Methodology/User centred approach

Given the above arguments, the research developed and validated a design method for diagram designers. Mathematical Diagram Design Method (MDDM) was iteratively developed and refined with users. It was also argued that Year Five (aged 9-11) is an important stage in developing an understanding of Shape and Space, (which is the foundation of Geometry) [9, 10]. Hence, during the course of developing MDDM a set of diagrams to teach Shape and Space to Year 5 was developed. The research focuses on collecting evidence from investigations with users (i.e. teachers, students educational publishers and designers) in order that the designer can have detailed knowledge about the **T**ask, **U**ser, **T**ool, and **E**nvironment (see Table 1), that can be used to inform the concept designs. We would argue that effective diagrams can only be produced by the application or understanding of knowledge from each category. Additionally, the designer is a factor that contributes to the quality of diagram. By engaging with designer, the user can assess the designer knowledge in the domain (mathematics). Such involvement minimises and eradicates errors early in the process.

Table 1: TUTED as applied to diagram design

Category	Detail	Knowledge acquisition
Task	Learning key concepts Measurable through achievement of learning outcomes	Understanding the curriculum (e.g. in terms of what has been learnt and needs to be learnt), how the concept is attained. Accessible through interviews with teachers and curriculum.
User	Cognitive development (in this case mathematical reasoning ability), perception of representations	Cognitive development and reasoning skills (e.g. Van Hiele and Piaget). Perception (especially Gestalt psychology). Accessible through literature review and discussion with teachers.
Tool	Visual representations e.g. static diagrams, animations and virtual reality.	Principles of diagrammatic and information design.
Environment	The classroom and way in which diagrams support learning	Observation of lessons. Interviews with teachers. Understanding of educational theories and trends.
Designer	Designer's knowledge in the domain, skill and competency	Through engaging with teachers, observing schools and students.

This approach, and the final MDDM was developed using an action learning approach, centred on three studies, as outlined below.

2.1 Field study

Observations and interviews with teachers and children identified curriculum requirements, current teaching methods, the role of diagrams in teaching, and the benefits and problems learners and teachers have with diagrams. The results indicated that: a) inaccurate diagrams are used in schools; b) such diagrams confuse children when they are struggling to learn new concepts; and c) teacher is often unable to guide student knowledge and development using current diagrams.

2.2 Evaluation of current diagrams learning

This stage involved an evaluation of existing textbook diagrams used to teach concepts of Shape and Space with students and designers. In respect to evaluation with students, Computer Assisted Learning (CAL) software was developed as a vehicle for presenting children with diagrammatic representations of existing materials. Pre and post test evaluation focused on the extent to which the current schoolbook diagrams supported/facilitated the learning of mathematical concepts. A simple paired-samples, *t*-test was carried out. The results indicated that the schoolbook diagrams did not improve students' learning significantly ($t\ (8) = -0.82$, $p= 0.44 >0.05$). These results provided a benchmark for testing a new method for diagram design. If the method has value, we would expect the diagrams developed using it to show an improvement over the performance of those used in the sample textbook.

In relation to understanding the current design process, five UK professional educational designers from four companies (with a mean range of 18 years experience), were interviewed to determine how diagrams for use in the teaching of mathematics are currently designed. Typically, designers followed the draft diagrams provided by the textbook author and embellished them following recommendations from the marketing department. No discussion occurred between the designer and author during early production; neither was there any evaluation with students or teachers (unless it was major publication) to ascertain the effectiveness of the diagram in supporting learning in different learning contexts. This lack of evaluation could be one of the reasons why erroneous diagrams are still found.

2.3 Development of a user centred design method to produce effective diagrams

The third stage involved the development and testing of a new method for diagram design. This involves two stages: 1) an establishment of diagram design principles abstracted from psychological and pedagogic literature; and 2) embedding these guidelines in an overall iterative design method.

2.3.1 Six design principles

1. *Salience* involves the ability of the design to gain the users' attention to the important features of the diagram, augmented by Gestalt principles, visual properties and motion [14]. For example, users will pay more attention to moving or dominant groups of objects.
2. *Mapping* entails the process of relating mathematical and graphical representations in a systematic manner. The learner may have difficulty in connecting the diagram to the domain being modeled. A viewer of the diagram must determine what each graphical element shown in the diagram represents and link those graphical elements to their experience to recognize the information. This may be different [14] from the way designer intended. For example, we learn the concept of fraction through the presentation of 'pie or cake segments'.
3. *Causality* requires the process of presenting graphical elements in relation to the mathematical state it represents. The graphic representation must portray the relationship between the graphical parts in time and space to reinforce cause and effect relationships. The principle is based on perceptual psychology [6]. For example, rotational transformation may be illustrated by an object moving in a circle around a pivot point accompanied concurrently by text showing the increasing degree size.
4. *Visual reinforcement* involves the repetition of the basic concept before advancing to a more difficult one. This principle is based on teaching principles of "Reinforcement' and 'Scaffolding' [1, 5]. Mathematical diagrams need to support this especially for young learners. For example, in teaching the concept of a cube, an underlying concept is the concept of a 'square' and that a cube has six square polygons.

The diagram needs to portray properties of 'square' first, which is the foundation before moving on to teaching the number of polygon.

5. *Mathematical generalisation presentation* involves the re-presentation of mathematical processes and their relationships [10, 17]. This principle is specific to mathematical teaching and learning. For example, to teach Shape and Space, the diagram should encourage the display of related properties between 2D and 3D geometry so that students can practice with reasoning about 2D and 3D properties and relations, i.e. a square-based pyramid, the diagram should display the 2D square's properties and how various properties such as volume, faces and edges make a 3D pyramid.

6. *Sequential ordered representation* involves the cueing and organisation of mathematical information, which underlines the mathematical generalisation. For example, the diagrammatic representation must depict the procedure and subgoal(s) of the concept following the logical process of the concept (e.g. a procedure of constructing a geometrical shape, distinctive properties or relationships of geometrical shapes). The principle is useful for designing procedural tasks such as mathematics, physics and furniture assembly. The principle is emphasized by cognitive scientists and mathematics teachers [2].

2.3.2 Developing MDDM

Adopting an action learning approach, we redesigned the existing diagrams, and reflected on this process to create a general diagram design method that would be user centred, iterative and support implicitly the designers' knowledge to produce effective diagrams. MDDM can be broken down into the following steps. Each stage was underpinned by theoretical guidelines, the six design principles and mediated by the experience of the domain experts (teachers) and feedback from the end users (children).

1. Clarification of the diagram requirements through discussion of the underlying concepts to be taught with teachers or other experts. Such discussion should be supported by existing visual representations where possible.
2. Evaluation of existing diagrams (in the case of redesign) by the teacher, to indicate real design problems and communicate these to the designer.

From these two stages a clear, written specification is generated which outlines the mathematical concepts to be delivered (e.g. equal sides and equal angles for an equilateral triangle), the failure of existing solutions and an indication of potential solutions. Additionally mutual understanding is built up – the designer acquires a greater understanding of the way in which the diagrams will be used and the concepts to be delivered. The teacher more fully understands the potential of the diagram.

3. Generation of concept designs, which are discussed with the teacher before detailed design and the development of key frames (for the animation). In this study, animation was used because it reduced the number of diagrammatic elements (arrows) and because time-based diagrams have been shown to reduce the perceived complexity of concepts.
4. Detail design and verification of animations, again the animation is designed iteratively in consultation with the teachers, who are asked to: 1) confirm that the diagram is necessary, 2) determine the appropriateness of the diagram, 3) reduce diagram complexity and confusion.

The iterative, formative development with the teachers was designed to assess the diagrams in terms of their delivery of key mathematical concepts, their likelihood of confusing the children and whether the design of the diagram's elements, keyframes and motion were appropriate (e.g. speed of animation, ambiguity and the accompanying text).

5. Formative evaluation with students in which the animations are presented one key frame at a time, in order to determine whether they are read as 'intended and the way the diagram extend geometrical understanding.

From this evaluation further refinements were made to the designs e.g. relating to font size and speed of presentation.

6. Summative evaluation with students (representatives of the end user population) to assess the extent to which the revised (or new) diagrams enhance learning. Two stages are involved in the summative evaluation of the final diagrams: 1) a pre and post-test measuring learning improvement, which designers can readily integrate within their design practice and 2) an interview investigating the way in which students construct

knowledge about key mathematical factors from the diagrams. Pairs of students were interviewed and Van Hiele's model of geometrical reasoning [19] applied to interpret the results (it should be noted that there are other models of analysis, which can be used such as the problem-solving ability level called Procept [20]). If the students could explain the mathematical properties in the animation, it meant the design for the property was successful.

3. Results and discussion

First, a comparative study was undertaken with two groups of matched students, a control group using standard material and an experimental one using the redesigned material. No difference in learning attainment was found between the two groups, however the reliability of this finding was suspect due to small sample sizes (n=10). Follow up interviews with five pairs of students and five teachers revealed that the diagrams still had design problems, but more importantly, showed that the new diagrams would potentially encourage greater engagement with the material and promote mathematical thinking.

Second, the method was formalized and explained to educational diagram developers. The developers felt that such a method would be of value to their practice, but expressed reservations about the time such a process would take and the need for statistical analysis. Obviously the adoption of a method entailing such a high degree of user involvement would require a major change in the industry. However, if more effective training material is to be produced such a change may be needed.

Third, a complimentary study was undertaken with teachers. Although the method had not been designed primarily for them, they would use the diagrams generated by MDDM, and could be involved with the evaluation or use the method to develop their own teaching material. The results confirmed that diagrams produced using MDDM would be valued, as they improved the presentation of geometrical information. They believed that the method could be useful in terms of problem-based design, creation of new mathematical diagrams, and error reduction. Furthermore, the life span of the diagram and textbook could be extended and hence justify the increased investment. The teachers also highlighted the need for more user centred design in the creation of learning material to take into account individual learning styles and the range of competencies in any one group.

4. Conclusion

The design process outlined above does not guarantee success. Rather, it provides the beginnings of a systematic and principled approach to the design of educational animations. It is claimed that this research contributes to knowledge by advancing methods for diagram design. Specifically, the method provides a stakeholder approach to design giving teachers and students input to the process so it based on user problems and needs. These stakeholder ideas can lead to alternative diagrams and improve the effectiveness of diagrams. The method provides design principles for designers based on the cognitive psychology and educational literature. The principles focus on presenting differences and similarities between mathematical concepts, causality, mapping, salience, and layout that can be applied for communicating mathematical entity relationship information and promoting students' mathematical reasoning. The evaluation method's validation was performed in two stages. The first stage evaluated the learning improvement quantitatively, while the second stage evaluated students' interpretation of diagrams using interview and discussion. This evaluation method is particularly important in publishing, where diagrams have sometimes been arbitrarily developed without evaluation. Designers evaluating new diagram designs can use a procedure similar to this.

Although the method is usable by designers we realize that there are limitations in this research. Further research needs to be carried out with a larger sample size with more diagrams and students and a professional designer should test the method in a professional context.

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